CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL

2016 Trial Examination

FORM VI

MATHEMATICS EXTENSION 1

Friday 12th August 2016

General Instructions

- Writing time — 2 hours
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 70 Marks

- All questions may be attempted.

Section I – 10 Marks

- Questions 1–10 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II – 60 Marks

- Questions 11–14 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your candidate number on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your candidate number on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Eleven.

Checklist

- SGS booklets — 4 per boy
- Multiple choice answer sheet
- Candidature — 109 boys

Examiner

SO

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

What are the asymptotes of $y = \dfrac{2x}{(x+3)(x-1)}$?

(A) $y = 0, \quad x = 1, \quad x = -3$

(B) $y = 0, \quad x = -1, \quad x = 3$

(C) $y = 2, \quad x = 1, \quad x = -3$

(D) $y = 2, \quad x = -1, \quad x = 3$

QUESTION TWO

Determine $\lim\limits_{x \to 0} \left(\dfrac{\sin x}{3 \tan x} \right)$.

(A) 0

(B) $\dfrac{1}{3}$

(C) 1

(D) 3

QUESTION THREE

What is the domain of $f(x) = e^{-\frac{1}{x}}$?

(A) $x > 0$

(B) $x \geq 0$

(C) $x \neq 0$

(D) all real x

QUESTION FOUR

What is the value of $\sin\left(\tan^{-1} a\right)$?

(A) $\dfrac{a}{\sqrt{1-a^2}}$

(B) $\dfrac{1}{\sqrt{1-a^2}}$

(C) $\dfrac{1}{\sqrt{1+a^2}}$

(D) $\dfrac{a}{\sqrt{1+a^2}}$

Examination continues next page ...

QUESTION FIVE

The monic quadratic equation with roots $m + n$ and $m - n$ is:

(A) $x^2 - 2mx + m^2 - n^2 = 0$

(B) $x^2 + 2mx + m^2 - n^2 = 0$

(C) $x^2 - 2mx + n^2 - m^2 = 0$

(D) $x^2 + 2mx + n^2 - m^2 = 0$

QUESTION SIX

A function is defined by the following rule:

$$f(x) = \begin{cases} \sin^{-1} x, & \text{for } -1 \le x < 0 \\ \cos^{-1} x, & \text{for } 0 \le x \le 1 \end{cases}$$

What is the value of $f\left(-\frac{1}{2}\right) + f(0)$?

(A) $-\dfrac{\pi}{6}$

(B) $\dfrac{\pi}{6}$

(C) $\dfrac{\pi}{3}$

(D) $\dfrac{2\pi}{3}$

QUESTION SEVEN

The range R of any particle projected from a point on a level plane at an angle of α to the horizontal with initial speed v is given by $R = \dfrac{v^2 \sin 2\alpha}{g}$.

A particle is projected at 50° to the horizontal. What other angle of projection would give the same range for this particle?

(A) 25°

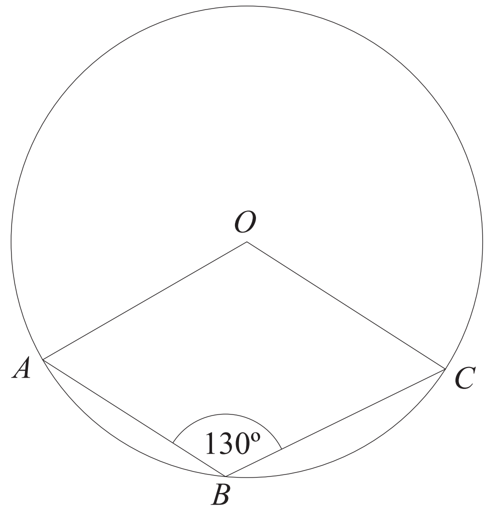
(B) 40°

(C) 80°

(D) 100°

Examination continues overleaf ...

QUESTION EIGHT



The points A, B and C lie on a circle with centre O. If $\angle ABC = 130°$, what is the size of $\angle AOC$?

(A) $50°$

(B) $65°$

(C) $100°$

(D) $260°$

QUESTION NINE

A particle is moving in simple harmonic motion with period 4 and amplitude 3. Which of the following is a possible equation for the velocity of the particle?

(A) $v = \dfrac{3\pi}{2} \cos \dfrac{\pi t}{2}$

(B) $v = 3 \cos \dfrac{\pi t}{2}$

(C) $v = \dfrac{3\pi}{4} \cos \dfrac{\pi t}{4}$

(D) $v = 3 \cos \dfrac{\pi t}{4}$

QUESTION TEN

Which of the following is a necessary condition if $a^2 > b^2$?

(A) $a > b$

(B) $a < b < 0$

(C) $a > 0 > b$

(D) $|a| > |b|$

End of Section I

Examination continues next page ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION ELEVEN (15 marks) Use a separate writing booklet. **Marks**

(a) Find the value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$. **1**

(b) Let $A = (4, -3)$ and $B = (8, 5)$. The interval AB is divided internally in the ratio $3:1$ by the point $P(x, y)$. Find the values of x and y. **2**

(c) Solve $\dfrac{5}{3x-2} > 2$. **3**

(d) The acute angle between the two lines $y = \dfrac{1}{2}x + 1$ and $y = mx + 3$ is $\dfrac{\pi}{4}$. Find all possible values of the constant m. **3**

(e) Find the general solution of $\cos 2x - \cos x = 2$. **3**

(f) Find the term independent of x in the expansion of $\left(x^2 - \dfrac{2}{x}\right)^{12}$. **3**

QUESTION TWELVE (15 marks) Use a separate writing booklet. **Marks**

(a) The polynomial $P(x) = 2x^3 + x^2 + ax + 6$ has a zero at $x = 2$.

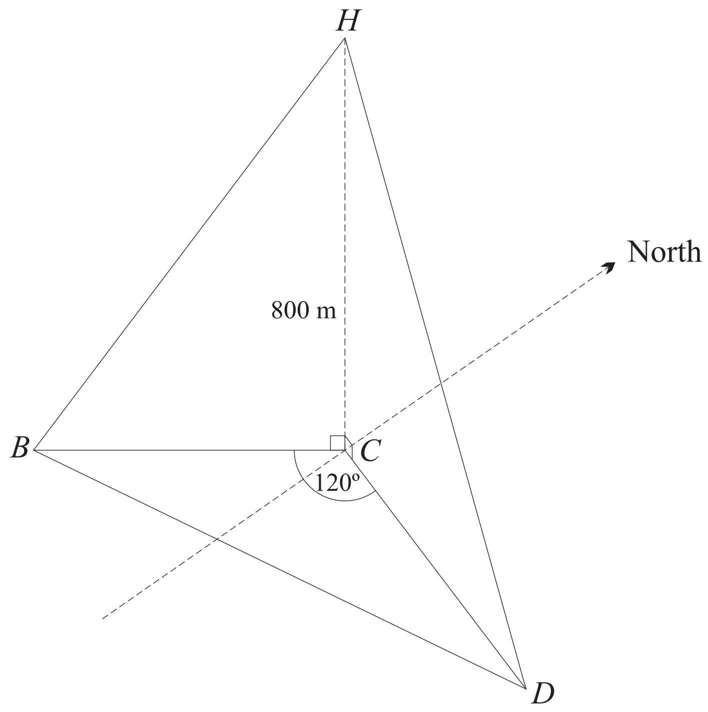
(i) Determine the value of a. **1**

(ii) Find the linear factors of $P(x)$. **2**

(iii) Hence, or otherwise, solve $P(x) \geq 0$. **1**

(b) Integrate $\displaystyle\int_0^{\frac{1}{\sqrt{3}}} \frac{\sin\left(\tan^{-1} x\right)}{1 + x^2}\,dx$ using the substitution $u = \tan^{-1} x$. **3**

(c) **2**

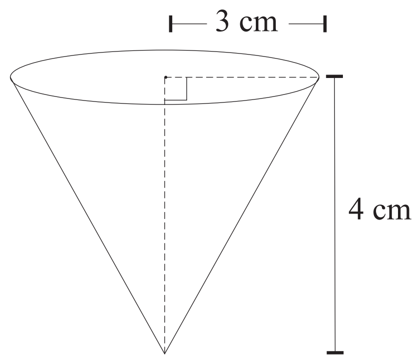


The diagram above shows a hot air balloon at point H with altitude 800 m. The passengers in the balloon can see a barn and a dam below, at points B and D respectively. Point C is directly below the hot air balloon. From the hot air balloon's position, the barn has a bearing of $250°$ and the dam has a bearing of $130°$, and $\angle BCD = 120°$. The angles of depression to the barn and the dam are $50°$ and $30°$ respectively.

How far is the barn from the dam, to the nearest metre?

(d) Prove by induction that $(x + y)$ is a factor of $x^{2n} - y^{2n}$, for all integers $n \geq 1$. **3**

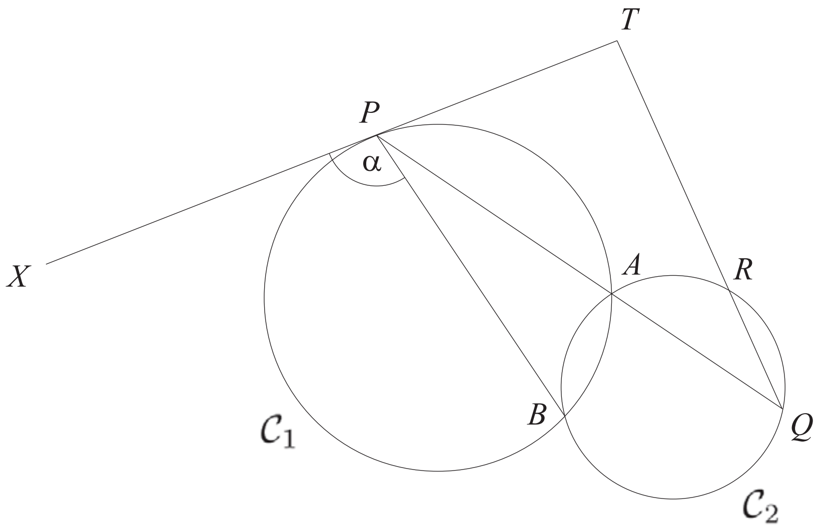
(e) **3**



The diagram above shows a vessel in the shape of a cone of radius 3 cm and height 4 cm. Water is poured into it at the rate of $10\,\text{cm}^3/\text{s}$. Find the rate at which the water level is rising when the depth is 2 cm.

QUESTION THIRTEEN (15 marks) Use a separate writing booklet. **Marks**

(a)



Two circles $\mathcal{C}_1$ and $\mathcal{C}_2$ intersect at A and B. A line through A meets the circles at P and Q respectively. A tangent is drawn from an external point T to touch the circle $\mathcal{C}_1$ at P. The line TQ intersects $\mathcal{C}_2$ at R.

(i) Given $\angle XPB = \alpha$, show that $\angle BRQ = 180^\circ - \alpha$, giving reasons. **2**

(ii) Hence show that $PTRB$ is a cyclic quadrilateral. **1**

(b) Consider the parabola $x^2 = 4ay$ with focus S. The normal at $P(2ap, ap^2)$ meets the y-axis at R and $\triangle SPR$ is equilateral.

(i) Show that the equation of the normal at P is $x + py = 2ap + ap^3$. **1**

(ii) Write down the coordinates of R. **1**

(iii) Prove that SP is equal in length to the latus rectum, that is $4a$ units. **3**

(c) (i) Show that $\dfrac{d}{dx}(x \ln x) = 1 + \ln x$. **1**

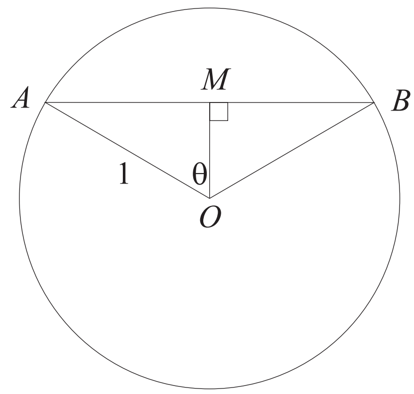
(ii) A particle is moving in a straight line. At time t seconds its position is x cm and its velocity is v cm/s. Initially $x = 1$ and $v = 2$. The acceleration a of the particle is given by the equation

$$a = 1 + \ln x.$$

Find the velocity v in terms of x. Be careful to explain why v is always positive. **2**

Examination continues next page ...

(d)



The circle above has radius 1 unit and the major arc joining A and B is twice as long as the chord AB. The point M lies on AB such that $AB \perp OM$. Let $\angle AOM = \theta$ where $0 < \theta < \dfrac{\pi}{2}$.

(i) Show that the length of the major arc satisfies the equation **1**

$$\pi - \theta = 2\sin\theta.$$

(ii) Let $\theta_0 \doteqdot 1{\cdot}5$ be a first approximation of θ. Use two applications of Newton's method to find a better approximation of θ. **2**

(iii) Use your answer to part (ii) to find the approximate length of the chord AB. **1**

Examination continues overleaf ...

QUESTION FOURTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) The mass M of a radioactive isotope is given by the equation $M = M_0 e^{-kt}$, where M_0 is the initial mass and k is a constant. The mass satisfies the equation $\dfrac{dM}{dt} = -kM$.

(i) If the half-life of this radioactive isotope is T, show that $k = \dfrac{\log_e 2}{T}$. **1**

(ii) A naturally occurring rock contains two radioactive isotopes X and Y. The half-lives of isotope X and isotope Y are T_X and T_Y respectively, where $T_X > T_Y$. Initially the mass of isotope Y is four times that of isotope X. **3**

Show that the rock will contain the same mass of both isotopes at time

$$\frac{2T_X T_Y}{T_X - T_Y}.$$

(b) Sketch the graph of $y = \dfrac{|x|}{x}$. **1**

(c) Consider the function $f(x) = \sin^{-1}\left(\dfrac{x^2-1}{x^2+1}\right)$.

(i) Find the domain of $f(x)$. **1**

(ii) Show that $f'(x) = \dfrac{2x}{|x|(x^2+1)}$. **2**

(iii) Determine the values of x for which $f(x)$ is increasing. **1**

(iv) Using part (b), explain the behaviour of $f'(x)$ as $x \to 0^+$ and $x \to 0^-$. **1**

(v) Draw a neat sketch of $y = f(x)$, indicating any intercepts with the axes and any asymptotes. **2**

(vi) Give the largest possible domain containing $x = 1$ for which $f(x)$ has an inverse function. Let this inverse function be $f^{-1}(x)$. **1**

(vii) Sketch $y = f^{-1}(x)$ on your original graph. **1**

(viii) Find the equation of $f^{-1}(x)$. **1**

End of Section II

END OF EXAMINATION

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- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ○ B ○ C ○ D ○

Question Two

A ○ B ○ C ○ D ○

Question Three

A ○ B ○ C ○ D ○

Question Four

A ○ B ○ C ○ D ○

Question Five

A ○ B ○ C ○ D ○

Question Six

A ○ B ○ C ○ D ○

Question Seven

A ○ B ○ C ○ D ○

Question Eight

A ○ B ○ C ○ D ○

Question Nine

A ○ B ○ C ○ D ○

Question Ten

A ○ B ○ C ○ D ○

EXTENSION MATHEMATICS SOLUTIONS

MULTIPLE CHOICE

1. Vertical asymptotes : $(x+3)(x-1)=0$

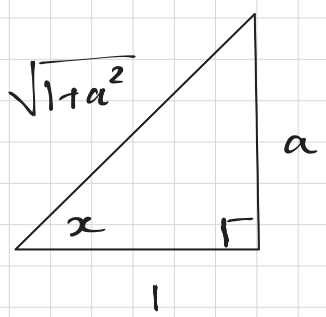
$x=-3$ and $x=1$ **A**

Horizontal asymptote: $y=0$

2. $\lim_{x \to 0} \frac{\sin x}{x} \times \frac{x}{\tan x} \times \frac{1}{3} = \frac{1}{3}$ **B**

3. $f(x) = e^{-\frac{1}{x}} \quad x \neq 0$ **C**

4.



Let $x = \tan^{-1} a$ **D**

$\tan x = a$

$\sin x = \frac{a}{\sqrt{1+a^2}}$

5. $(x-(m+n))(x-(m-n)) = 0$ **A**

$x^2 - 2mx + m^2 - n^2 = 0$

6. $f(-\frac{1}{2}) + f(0)$

$= \sin^{-1}(-\frac{1}{2}) + \cos^{-1}(0)$

$= -\frac{\pi}{6} + \frac{\pi}{2}$ **C**

$= \frac{\pi}{3}$

7. $\sin(2 \times 50^\circ) = \sin 100^\circ = \sin 80^\circ = \sin(2 \times 40^\circ)$ **B**

8. Reflex $\angle AOC = 260^\circ$ (angle at the centre is twice the angle at the circumference)

$\therefore \angle AOC = 100^\circ$ **C**

9. $T = \frac{2\pi}{n}$

$= \frac{\pi}{2}$

$x = 3\sin\frac{\pi t}{2}$

$v = \frac{3\pi}{2}\cos\frac{\pi t}{2}$

A

10. A sufficient

B neither necessary nor sufficient

C sufficient

D necessary and sufficient

1	2	3	4	5	6	7	8	9	10
A	B	C	D	A	C	B	C	A	D

QN 11

(a) $\sin^{-1}\left(\sin\left(\frac{3\pi}{5}\right)\right)$

$= \sin^{-1}\left(\sin\frac{2\pi}{5}\right)$

$= \frac{2\pi}{5}$ ✓

(b) A B

$(4,-3)$ $(8,5)$

3 : 1

$x = \frac{3\times 8 + 1\times 4}{4}$ $y = \frac{3\times 5 - 1\times 3}{4}$

$= \frac{28}{4}$ $= \frac{12}{4}$

$= 7$ ✓ $= 3$ ✓

So $(x,y) = (7,3)$

(c) $\frac{5}{3x-2} > 2$

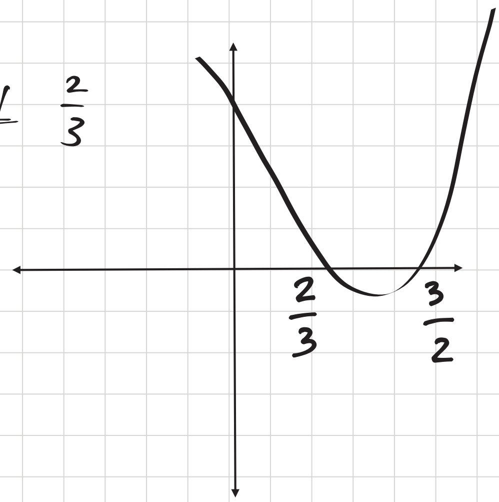
$5(3x-2) > 2(3x-2)^2$ ✓ $x \neq \frac{2}{3}$

$2(3x-2)^2 - 5(3x-2) < 0$

$(3x-2)(6x-9) < 0$ ✓

$3(3x-2)(2x-3) < 0$

So $\frac{2}{3} < x < \frac{3}{2}$ ✓



(d) $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$1 = \left|\dfrac{m_1 - \frac{1}{2}}{1 + \frac{m_1}{\frac{1}{2}}}\right|$ ✓

$1 + \dfrac{m_1}{2} = m_1 - \dfrac{1}{2}$ or $1 + \dfrac{m_1}{2} = -m_1 + \dfrac{1}{2}$

$-\dfrac{m_1}{2} = -\dfrac{3}{2}$ $\qquad \dfrac{3m_1}{2} = -\dfrac{1}{2}$

$m_1 = 3$ ✓ $\qquad m_1 = -\dfrac{1}{3}$ ✓

So the possible values of the constant m are 3 and $-\frac{1}{3}$.

(e) $\cos 2x - \cos x = 2$

$2\cos^2 x - 1 - \cos x - 2 = 0$ ✓

$2\cos^2 x - \cos x - 3 = 0$

$(2\cos x - 3)(\cos x + 1) = 0$

$\cos x = \dfrac{3}{2}$ $\qquad \cos x = -1$ ✓

$\downarrow$

no solution $\qquad x = -\pi + 2n\pi$, where n is an integer ✓

(f) Let the term be ${}^{12}C_k (x^2)^{12-k}\left(-\frac{2}{x}\right)^k$

$= {}^{12}C_k \times x^{24-2k} \times (-1)^k \times 2^k \times x^{-k}$

$= {}^{12}C_k \times x^{24-3k} \times (-1)^k \times 2^k$ ✓

For the term independent of x, $24 - 3k = 0$

$k = 8$ ✓

So ${}^{12}C_8 \times x^0 \times (-1)^8 \times 2^8$

$= {}^{12}C_8 \times 2^8$ ✓

$= 495 \times 256$

$= 126\,720$

QN 12

(a)(i) $P(2) = 0$

$0 = 16 + 4 + 2a + 6$

$2a = -26$

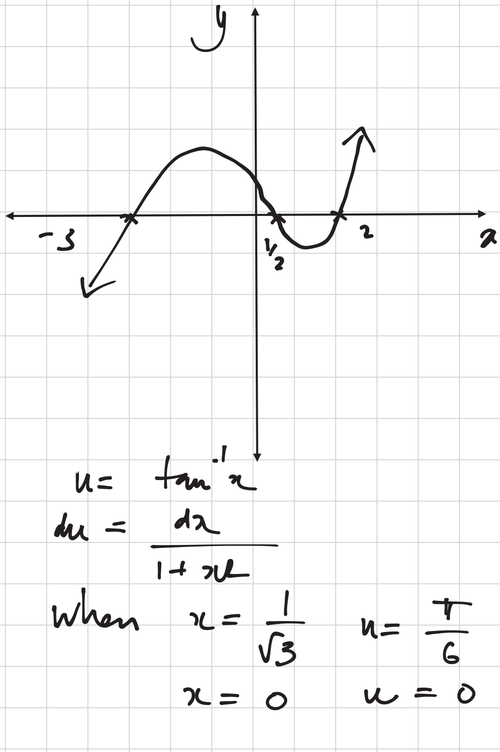
$a = -13$ ✓

(ii)

$$\begin{array}{r} 2x^2 + 5x - 3 \\ x-2 \overline{\smash{)}\, 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 - 4x^2} \quad\quad\quad \\ 5x^2 - 13x \quad \\ \underline{5x^2 - 10x} \quad \\ -3x + 6 \\ -3x + 6 \end{array}$$ ✓

$2x^2 + 5x - 3 = (2x-1)(x+3)$

so $P(x) = (x-2)(2x-1)(x+3)$ ✓



(iii) $P(x) \geq 0$

$-3 \leq x \leq \frac{1}{2}$ or $x \geq 2$ ✓

(b) $\displaystyle\int_0^{\frac{1}{\sqrt{3}}} \frac{\sin(\tan^{-1}x)}{1+x^2}\,dx$

$u = \tan^{-1}x$

$du = \dfrac{dx}{1+x^2}$

when $x = \dfrac{1}{\sqrt{3}}$ $\quad u = \dfrac{\pi}{6}$

$x = 0 \quad u = 0$

$= \displaystyle\int_0^{\frac{\pi}{6}} \sin u\,du$ ✓

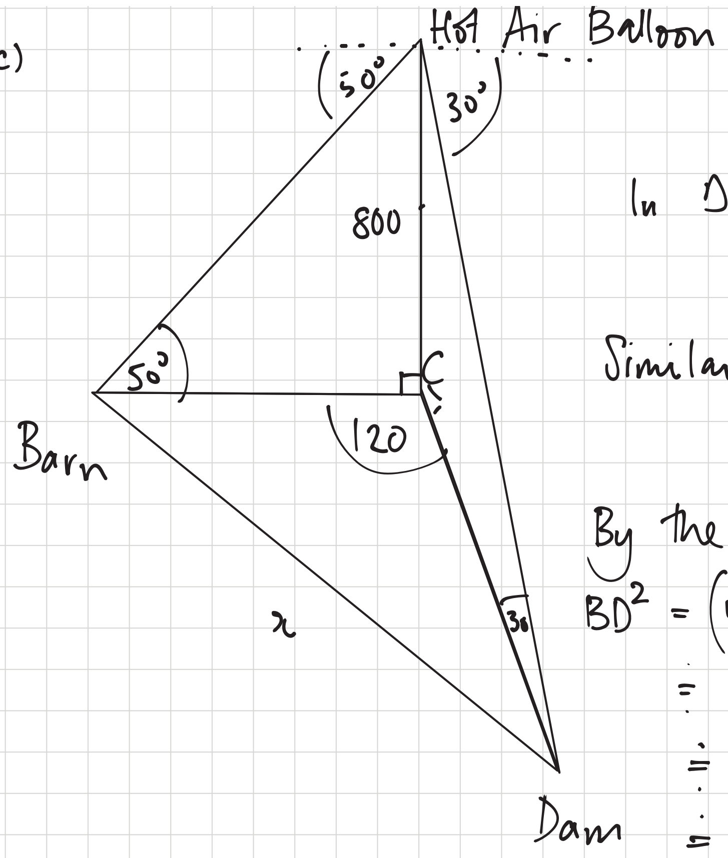
$= -\left[\cos u\right]_0^{\frac{\pi}{6}}$ ✓

$= -\left(\dfrac{\sqrt{3}}{2} - 1\right)$

$= 1 - \dfrac{\sqrt{3}}{2}$ ✓

$= \dfrac{2-\sqrt{3}}{2}$

(c)



In ΔBCH, $\tan 50° = \frac{800}{BC}$

$BC = 800 \cot 50°$

Similarly in ΔDCH, $\tan 30° = \frac{800}{DC}$

$DC = 800 \cot 30°$

By the cosine rule:

$BD^2 = (\cot^2 50° + \cot^2 30° - 2\cot 50° \cot 30° \cos 120°)$

$= 800^2 \times 5.157$

$\doteqdot 3300488$

$\doteqdot 1816.722\ldots$

$\doteqdot 1817m$

(d) Prove $x^{2n} - y^{2n}$ has $(x+y)$ as a factor for all integers $n \geq 1$.

① For $n=1$: $x^2 - y^2 = (x+y)(x-y)$, which has $(x+y)$ as a factor

② Assume true for $n=k$:

$x^{2k} - y^{2k} = m(x+y)$ ⊛ where m is an expression in x & y

③ Prove true for $n=k+1$:

From ⊛ $x^{2k} = m(x+y) + y^{2k}$

$x^{2k+2} - y^{2k+2}$

$= x \cdot x^{2k} - y \cdot y^{2k}$

$= x^2 [m(x+y) + y^{2k}] - y^2 y^{2k}$

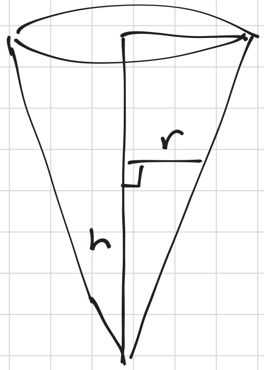
$= mx^2(x+y) + x^2 y^{2k} - y^2 \cdot y^{2k}$

$= mx^2(x+y) + y^{2k}(x+y)(x-y)$

$= x+y\, [mx^2 + y^{2k}(x-y)]$

So from parts (2) & (3) and by mathematical induction $(x+y)$ is a factor of $x^{2n}-y^{2n}$ for all integers $n \geq 1$.

(e)



$\frac{r}{h} = \frac{3}{4}$

$r = \frac{3h}{4}$

$V = \frac{1}{3}\pi\left(\frac{3h}{4}\right)^2 h$

$= \frac{3h^3\pi}{16}$ ✓

$\frac{dV}{dh} = \frac{9h^2\pi}{16}$

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$10 = \frac{9 \times 4 \times \pi}{16} \times \frac{dh}{dt}$ ✓

$\frac{dh}{dt} = \frac{40}{9\pi}$ cm/s ✓

So the water is rising at a rate of $\frac{40}{9\pi}$ cm/s when the depth of the cone is 2cm

Q13

(a)(i) $\angle XPB = \alpha$ Join Points A and B, and B and R.

$\angle PAB = \alpha$ (alternate segment theorem) ✓

$\angle BAQ = 180° - \alpha$ (straight line)

$\angle BRQ = 180° - \alpha$ (angles standing on the same arc) ✓

(ii) $\angle TRB = \alpha$ (straight angle)

$\angle XPT = \angle TRB$

∴ PTRB is a cyclic quadrilateral

(exterior angle is equal to the opposite interior angle) ✓

(b)(i) $\frac{dy}{dx} = \frac{2ap}{2a}$

$= p$

so gradient of the normal is $\frac{-1}{p}$

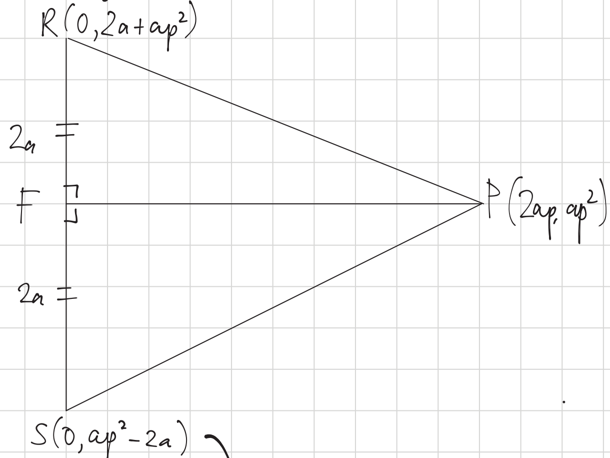
equation of the normal :

$$y - ap^2 = -\frac{1}{p}(x - 2ap)$$

$$py - ap^3 = -x + 2ap$$

$$x + py = 2ap + ap^3$$ ✓

(ii) $R(0, 2a + ap^2)$ ✓



(iii) Let F be foot of the perpendicular from P to the y-axis.

Then $FR = FS = 2a$ ✓

(ΔSRP is isosceles with altitude FP)

Hence $SR = 4a$ and since ΔSPR is equilateral

$SP = SR$
$= 4a$
$=$ length of latus rectum. ✓✓

(c) (i) $\frac{d}{dx}(x \ln x) = \ln x + \frac{1}{x} \times x$
$= \ln x + 1$ ✓

(ii) $a = 1 + \ln x$

$\frac{1}{2}v^2 = x \ln x + c$

at $v = 2$, $x = 1$ so $c = 2$

$\frac{1}{2}v^2 = x \ln x + 2$

$v^2 = 2x \ln x + 4$

$v = \pm\sqrt{2x \ln x + 4}$

$\ln x$ is always increasing so at $t = 0$, $a > 0$

so v is always positive ✓

$\therefore v = \sqrt{2x \ln x + 4}$ ✓

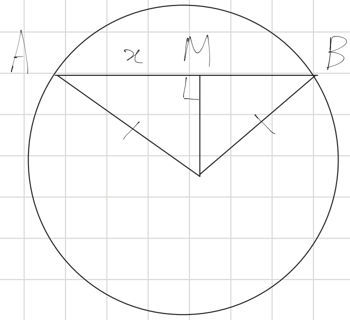
(d) (i) Let $AM = x$

chord $AB = 2x$
major arc $AB = 4x$

$x = \sin\theta$
so $AB = 2\sin\theta$

Major arc $AB = 2\pi - 2\theta$

so $2\pi - 2\theta = 4\sin\theta$
$\pi - \theta = 2\sin\theta$ ✓



(ii) $2\sin\theta + \theta - \pi = 0$

So $f(\theta) = 2\sin\theta + \theta - \pi$
$f'(\theta) = 2\cos\theta + 1$ ✓

$$\theta_1 = \theta_0 - \frac{f(\theta)}{f'(\theta)}$$

$$= 1.5 - \frac{(2\sin\theta + \theta - \pi)}{2\cos\theta + 1}$$

$= 1.19$

$\theta_2 = 1.24433\ldots$ ✓

(iii) $AB = 2\sin\theta$
$\doteqdot 2 \times \sin(1.24433\ldots)$
$\doteqdot 1.89436\ldots$ ✓

$\boxed{\text{Q14}}$

(a) (i) $\frac{1}{2}M_0 = M_0 e^{-kT}$

$$\frac{1}{2} = e^{-kT}$$

$$\ln\left(\frac{1}{2}\right) = -kT$$

$$k = \frac{\ln 2}{T} \quad \checkmark$$

(ii) $X = X_0 e^{-k_X t}$

$$Y = Y_0 e^{-k_Y t} = 4X_0 e^{-k_Y t}$$

$$4X_0 e^{-k_Y t} = X_0 e^{-k_X t} \quad \checkmark$$

$$\frac{4}{e^{k_Y t}} = \frac{1}{e^{k_X t}}$$

$$4 = \frac{e^{k_Y t}}{e^{k_X t}}$$

$$4 = e^{t(k_Y - k_X)}$$

$$2\ln 2 = t(k_Y - k_X)$$

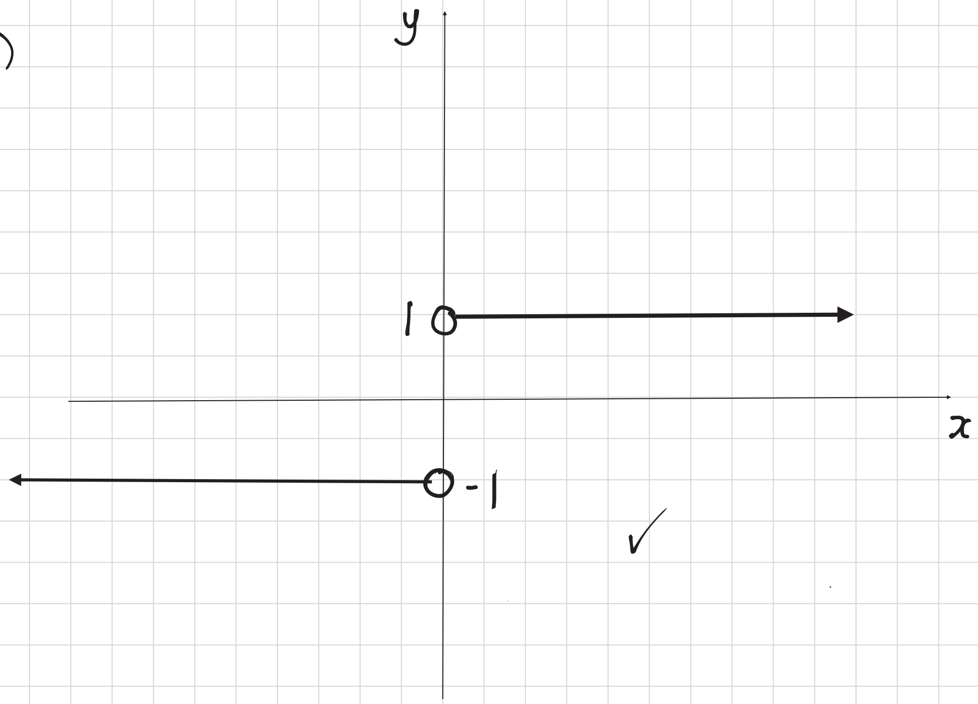
$$2\ln 2 = t\left(\frac{\ln 2}{T_Y} - \frac{\ln 2}{T_X}\right) \quad \checkmark$$

(using part (i))

$$2 = t\left(\frac{T_X - T_Y}{T_X T_Y}\right)$$

$$t = \frac{2T_X T_Y}{T_X - T_Y} \quad \checkmark$$

(b)



✓

(c) (i) $f(x) = \sin^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

Domain: $-1 \le \left(\frac{x^2-1}{x^2+1}\right) \le 1$

$-x^2-1 \le x^2-1 \le x^2+1$

$-x^2-1 \le x^2-1$ and $x^2-1 \le x^2+1$

$-2x^2 \le 0$ $\quad$ $0 \le 2$

$-x^2 \le 0$ $\quad$ true for all x

So domain is all real x ✓

(ii) Let $y = \sin^{-1}u$

$u = x^2-1 \quad v = x^2+1$

$u' = 2x \quad v' = 2x$

$\frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}$

$\frac{du}{dx} = \frac{2x(x^2+1)-2x(x^2-1)}{(x^2+1)^2}$

$= \frac{2x(x^2+1-x^2+1)}{(x^2+1)^2}$

$= \frac{4x}{(x^2+1)^2}$

$\frac{dy}{du} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{\sqrt{1-\left(\frac{x^2-1}{x^2+1}\right)^2}} \times \frac{4x}{(x^2+1)^2}$ ✓

$= \frac{(x^2+1) \times 4x}{\sqrt{x^4+2x^2+1-(x^4-2x^2+1)} \times (x^2+1)^2}$

$= \frac{4x}{\sqrt{4x^2} \times (x^2+1)}$

$$= \frac{4x}{2|x| \times (x^2+1)} \quad \left(\text{by definition } \sqrt{x^2} = |x|\right)$$

$$= \frac{2x}{|x| \times (x^2+1)}$$

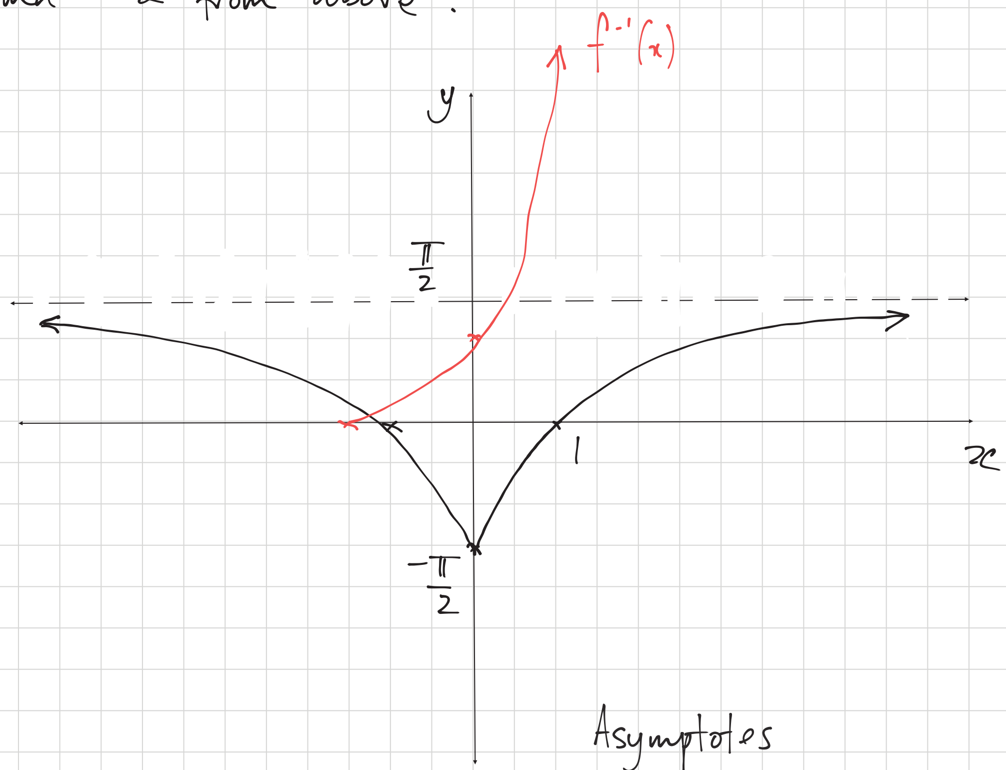
(iii) Increasing when $\frac{dy}{dx} > 0$

$x > 0$

(iv) $\lim_{x \to 0^+} f'(x) = 2$

$\lim_{x \to 0^-} f'(x) = -2$

The gradient approaches $+2$ from below and -2 from above.



x-intercept at $f(x) = 0$

$x^2 - 1 = 0$

$x = \pm 1$

Asymptotes

as $x \to \infty$ $f(x) \to \sin^{-1} 1$

$\to \frac{\pi}{2}$

as $x \to -\infty$ $f(x) \to \sin^{-1} 1$

$\to \frac{\pi}{2}$

Horizontal asymptote: $y = \frac{\pi}{2}$

(vi) $x \geqslant 0$ ✓

(vii) See graph ✓

(viii) $x = \sin^{-1}\left(\frac{y^2-1}{y^2+1}\right)$

$\sin x = \frac{y^2-1}{y^2+1}$

$\sin x(y^2+1) = y^2-1$

$y^2 \sin x + \sin x = y^2-1$

$y^2(1-\sin x) = 1+\sin x$

$y^2 = \frac{1+\sin x}{1-\sin x}$

$y = \sqrt{\frac{1+\sin x}{1-\sin x}}$ ✓

$\therefore f^{-1}(x) = \sqrt{\frac{1+\sin x}{1-\sin x}}$